Chapter 1 Page 1

Proportions

Write Ratios A **ratio** is a comparison of two quantities. The ratio a to b, where b is not zero, can be written as $\frac{a}{b}$ or $a{:}b$. The ratio of two quantities is sometimes called a **scale factor**. For a scale factor, the units for each quantity are the same.

Example 1 **In 2002, the Chicago Cubs baseball team won 67 games out of 162. Write a ratio for the number of games won to the total number of games played.**

To find the ratio, divide the number of games won by the total number of games played. The result is $\frac{67}{162}$, which is about 0.41. The Chicago Cubs won about 41% of their games in 2002.

Example 2 **A doll house that is 15 inches tall is a scale model of a real house with a height of 20 feet. What is the ratio of the height of the doll house to the height of the real house?**

To start, convert the height of the real house to inches.

20 feet $\times$ 12 inches per foot $=$ 240 inches

To find the ratio or scale factor of the heights, divide the height of the doll house by the height of the real house. The ratio is 15 inches:240 inches or 1:16. The height of the doll house is $\frac{1}{16}$ the height of the real house.

Use Properties of Proportions A statement that two ratios are equal is called a **proportion**. In the proportion $\frac{a}{b} = \frac{c}{d}$, where b and d are not zero, the values a and d are the **extremes** and the values b and c are the **means**. In a proportion, the product of the means is equal to the product of the extremes, so $ad = bc$.

$$\frac{a}{b} = \frac{c}{d}$$

$$\underset{\substack{\uparrow \\ \text{extremes}}}{a \cdot d} = \underset{\substack{\uparrow \\ \text{means}}}{b \cdot c}$$

Example 1 **Solve $\frac{9}{16} = \frac{27}{x}$.**

$\frac{9}{16} = \frac{27}{x}$

$9 \cdot x = 16 \cdot 27$ Cross products

$9x = 432$ Multiply.

$x = 48$ Divide each side by 9.

Example 2 **A room is 49 centimeters by 28 centimeters on a scale drawing of a house. For the actual room, the larger dimension is 14 feet. Find the shorter dimension of the actual room.**

If x is the room's shorter dimension, then

$\frac{28}{49} = \frac{x}{14}$ $\frac{\text{shorter dimension}}{\text{longer dimension}}$

$49x = 392$ Cross products

$x = 8$ Divide each side by 49.

The shorter side of the room is 8 feet.

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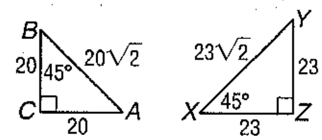
Similar Polygons

Identify Similar Figures

Example 1 **Determine whether the triangles are similar.**



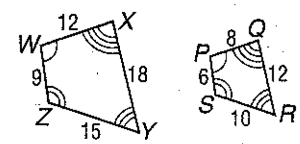
Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.

$\angle C \cong \angle Z$ because they are right angles, and $\angle B \cong \angle X$. By the Third Angle Theorem, $\angle A \cong \angle Y$.

For the sides, $\frac{BC}{XZ} = \frac{20}{23}$, $\frac{BA}{XY} = \frac{20\sqrt{2}}{23\sqrt{2}} = \frac{20}{23}$, and $\frac{AC}{YZ} = \frac{20}{23}$.

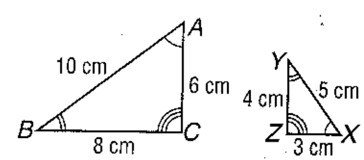
The side lengths are proportional. So $\triangle BCA \sim \triangle XZY$.

Example 2 **Is polygon $WXYZ \sim$ polygon $PQRS$?**

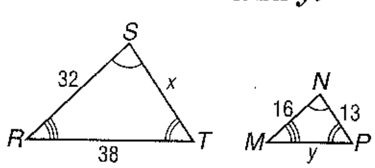


For the sides, $\frac{WX}{PQ} = \frac{12}{8} = \frac{3}{2}$, $\frac{XY}{QR} = \frac{18}{12} = \frac{3}{2}$, $\frac{YZ}{RS} = \frac{15}{10} = \frac{3}{2}$, and $\frac{ZW}{SP} = \frac{9}{6} = \frac{3}{2}$. So corresponding sides are proportional. Also, $\angle W \cong \angle P$, $\angle X \cong \angle Q$, $\angle Y \cong \angle R$, and $\angle Z \cong \angle S$, so corresponding angles are congruent. We can conclude that polygon $WXYZ \sim$ polygon $PQRS$.

Scale Factors When two polygons are similar, the ratio of the lengths of corresponding sides is called the **scale factor**. At the right, $\triangle ABC \sim \triangle XYZ$. The scale factor of $\triangle ABC$ to $\triangle XYZ$ is 2 and the scale factor of $\triangle XYZ$ to $\triangle ABC$ is $\frac{1}{2}$.



Example 1 **The two polygons are similar. Find x and y.**



Use the congruent angles to write the corresponding vertices in order.

$\triangle RST \sim \triangle MNP$

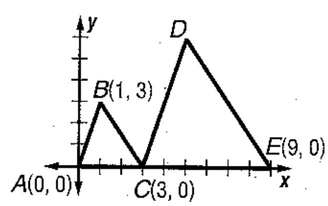
Write proportions to find x and y.

$\frac{32}{16} = \frac{x}{13}$ $\qquad$ $\frac{38}{y} = \frac{32}{16}$

$16x = 32(13)$ $\qquad$ $32y = 38(16)$

$x = 26$ $\qquad$ $y = 19$

Example 2 **$\triangle ABC \sim \triangle CDE$. Find the scale factor and find the lengths of $\overline{CD}$ and $\overline{DE}$.**



$AC = 3 - 0 = 3$ and $CE = 9 - 3 = 6$. The scale factor of $\triangle CDE$ to $\triangle ABC$ is 6:3 or 2:1. Using the Distance Formula, $AB = \sqrt{1 + 9} = \sqrt{10}$ and $BC = \sqrt{4 + 9} = \sqrt{13}$. The lengths of the sides of $\triangle CDE$ are twice those of $\triangle ABC$, so $DC = 2(BA)$ or $2\sqrt{10}$ and $DE = 2(BC)$ or $2\sqrt{13}$.

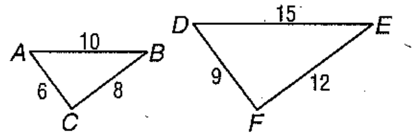
Similar Triangles

Identify Similar Triangles

Here are three ways to show that two triangles are similar.

AA Similarity	Two angles of one triangle are congruent to two angles of another triangle.
SSS Similarity	The measures of the corresponding sides of two triangles are proportional.
SAS Similarity	The measures of two sides of one triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent.

Example 1 **Determine whether the triangles are similar.**



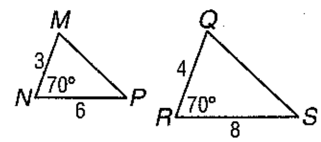
$\frac{AC}{DF} = \frac{6}{9} = \frac{2}{3}$

$\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$

$\frac{AB}{DE} = \frac{10}{15} = \frac{2}{3}$

$\triangle ABC \sim \triangle DEF$ by SSS Similarity.

Example 2 **Determine whether the triangles are similar.**



$\frac{3}{4} = \frac{6}{8}$, so $\frac{MN}{QR} = \frac{NP}{RS}$.

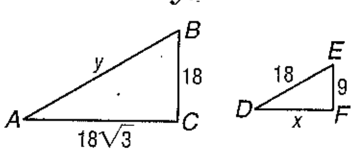
$m\angle N = m\angle R$, so $\angle N \cong \angle R$.

$\triangle NMP \sim \triangle RQS$ by SAS Similarity.

Use Similar Triangles

Similar triangles can be used to find measurements.

Example 1 $\triangle ABC \sim \triangle DEF$. **Find x and y.**



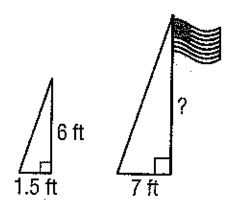
$$\frac{AC}{DF} = \frac{BC}{EF} \qquad \frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{18\sqrt{3}}{x} = \frac{18}{9} \qquad \frac{y}{18} = \frac{18}{9}$$

$$18x = 9(18\sqrt{3}) \qquad 9y = 324$$

$$x = 9\sqrt{3} \qquad y = 36$$

Example 2 **A person 6 feet tall casts a 1.5-foot-long shadow at the same time that a flagpole casts a 7-foot-long shadow. How tall is the flagpole?**



The sun's rays form similar triangles.
Using x for the height of the pole, $\frac{6}{x} = \frac{1.5}{7}$,
so $1.5x = 42$ and $x = 28$.
The flagpole is 28 feet tall.

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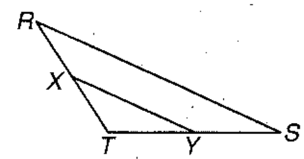
Parallel Lines and Proportional Parts

Proportional Parts of Triangles In any triangle, a line parallel to one side of a triangle separates the other two sides proportionally. The converse is also true.



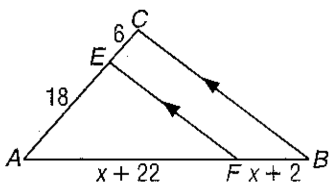
If X and Y are the midpoints of $\overline{RT}$ and $\overline{ST}$, then $\overline{XY}$ is a **midsegment** of the triangle. The Triangle Midsegment Theorem states that a midsegment is parallel to the third side and is half its length.

If $\overleftrightarrow{XY} \parallel \overleftrightarrow{RS}$, then $\frac{RX}{XT} = \frac{SY}{YT}$.

If $\frac{RX}{XT} = \frac{SY}{YT}$, then $\overleftrightarrow{XY} \parallel \overleftrightarrow{RS}$.

If $\overline{XY}$ is a midsegment, then $\overleftrightarrow{XY} \parallel \overleftrightarrow{RS}$ and $XY = \frac{1}{2}RS$.

Example 1 **In $\triangle ABC$, $\overline{EF} \parallel \overline{CB}$. Find x.**



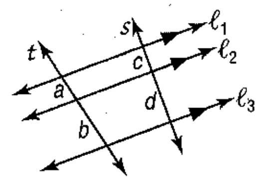
Since $\overline{EF} \parallel \overline{CB}$, $\frac{AF}{FB} = \frac{AE}{EC}$.

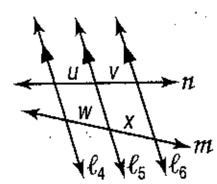
$$\frac{x + 22}{x + 2} = \frac{18}{6}$$
$$6x + 132 = 18x + 36$$
$$96 = 12x$$
$$8 = x$$

Example 2 **A triangle has vertices $D(3, 6)$, $E(-3, -2)$, and $F(7, -2)$. Midsegment $\overline{GH}$ is parallel to $\overline{EF}$. Find the length of $\overline{GH}$.**

$\overline{GH}$ is a midsegment, so its length is one-half that of $\overline{EF}$. Points E and F have the same y-coordinate, so $EF = 7 - (-3) = 10$. The length of midsegment $\overline{GH}$ is 5.

Divide Segments Proportionally When three or more parallel lines cut two transversals, they separate the transversals into proportional parts. If the ratio of the parts is 1, then the parallel lines separate the transversals into congruent parts.





If $\ell_1 \parallel \ell_2 \parallel \ell_3$, then $\frac{a}{b} = \frac{c}{d}$.

If $\ell_4 \parallel \ell_5 \parallel \ell_6$ and $\frac{u}{v} = 1$, then $\frac{w}{x} = 1$.

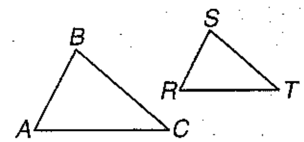
Example **Refer to lines ℓ_1, ℓ_2, and ℓ_3 above. If $a = 3$, $b = 8$, and $c = 5$, find d.**

$\ell_1 \parallel \ell_2 \parallel \ell_3$ so $\frac{3}{8} = \frac{5}{d}$. Then $3d = 40$ and $d = 13\frac{1}{3}$.

Chapter 7

Parts of Similar Triangles

Perimeters If two triangles are similar, their perimeters have the same proportion as the corresponding sides.



If $\triangle ABC \sim \triangle RST$, then

$$\frac{AB + BC + AC}{RS + ST + RT} = \frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}.$$

Example **Use the diagram above with $\triangle ABC \sim \triangle RST$. If $AB = 24$ and $RS = 15$, find the ratio of their perimeters.**

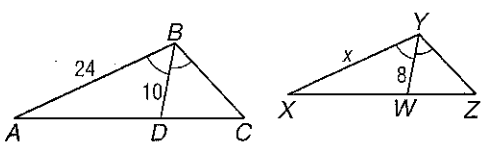
Since $\triangle ABC \sim \triangle RST$, the ratio of the perimeters of $\triangle ABC$ and $\triangle RST$ is the same as the ratio of corresponding sides.

Therefore $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle RST} = \frac{24}{15}$

$= \frac{8}{5}$

Special Segments of Similar Triangles When two triangles are similar, corresponding altitudes, angle bisectors, and medians are proportional to the corresponding sides. Also, in any triangle an angle bisector separates the opposite side into segments that have the same ratio as the other two sides of the triangle.

Example 1 **In the figure, $\triangle ABC \sim \triangle XYZ$, with angle bisectors as shown. Find x.**



Since $\triangle ABC \sim \triangle XYZ$, the measures of the angle bisectors are proportional to the measures of a pair of corresponding sides.

$\frac{AB}{XY} = \frac{BD}{YW}$

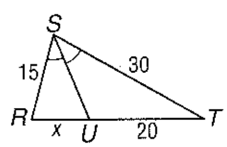
$\frac{24}{x} = \frac{10}{8}$

$10x = 24(8)$

$10x = 192$

$x = 19.2$

Example 2 **$\overline{SU}$ bisects $\angle RST$. Find x.**



Since $\overline{SU}$ is an angle bisector, $\frac{RU}{TU} = \frac{RS}{TS}$.

$\frac{x}{20} = \frac{15}{30}$

$30x = 20(15)$

$30x = 300$

$x = 10$