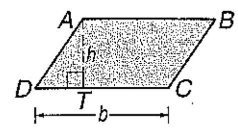
11-1

Areas of Parallelograms

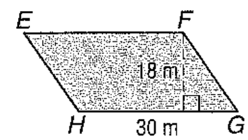
Areas of Parallelograms A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a **base**. Each base has a corresponding **altitude**, and the length of the altitude is the **height** of the parallelogram. The area of a parallelogram is the product of the base and the height.

Area of a Parallelogram	If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



The area of parallelogram $ABCD$ is $CD \cdot AT$.

Example **Find the area of parallelogram *EFGH*.**



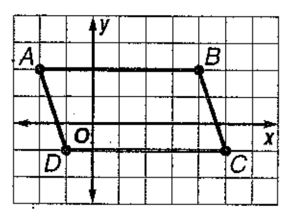
$A = bh$	Area of a parallelogram
$= 30(18)$	$b = 30$, $h = 18$
$= 540$	Multiply.

The area is 540 square meters.

Areas of Parallelograms

Parallelograms on the Coordinate Plane To find the area of a quadrilateral on the coordinate plane, use the Slope Formula, the Distance Formula, and properties of parallelograms, rectangles, squares, and rhombi.

Example **The vertices of a quadrilateral are $A(-2, 2)$, $B(4, 2)$, $C(5, -1)$, and $D(-1, -1)$.**



a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

Graph the quadrilateral. Then determine the slope of each side.

slope of $\overline{AB} = \frac{2 - 2}{4 - (-2)}$ or 0

slope of $\overline{CD} = \frac{-1 - (-1)}{-1 - 5}$ or 0

slope $\overline{AD} = \frac{2 - (-1)}{-2 - (-1)}$ or -3

slope $\overline{BC} = \frac{-1 - 2}{5 - 4}$ or -3

Opposite sides have the same slope. The slopes of consecutive sides are not negative reciprocals of each other, so consecutive sides are not perpendicular. $ABCD$ is a parallelogram; it is not a rectangle or a square.

b. Find the area of *ABCD*.

From the graph, the height of the parallelogram is 3 units and $AB = |4 - (-2)| = 6$.

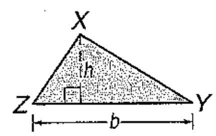
$A = bh$	Area of a parallelogram
$= 6(3)$	$b = 6$, $h = 3$
$= 18 \text{ units}^2$	Multiply.

11-2

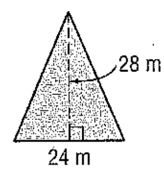
Areas of Triangles, Trapezoids, and Rhombi

Areas of Triangles The area of a triangle is half the area of a rectangle with the same base and height as the triangle.

If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then $A = \frac{1}{2}bh$.



Example **Find the area of the triangle.**



$A = \frac{1}{2}bh$ Area of a triangle

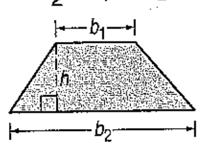
$= \frac{1}{2}(24)(28)$ $b = 24$, $h = 28$

$= 336$ Multiply.

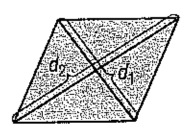
The area is 336 square meters.

Areas of Trapezoids and Rhombi The area of a trapezoid is the product of half the height and the sum of the lengths of the bases. The area of a rhombus is half the product of the diagonals.

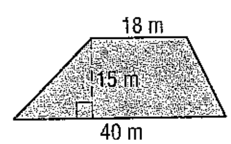
If a trapezoid has an area of A square units, bases of b_1 and b_2 units, and a height of h units, then $A = \frac{1}{2}h(b_1 + b_2)$.



If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$.



Example **Find the area of the trapezoid.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(15)(18 + 40)$ $h = 15$, $b_1 = 18$, $b_2 = 40$

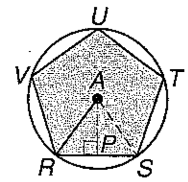
$= 435$ Simplify.

The area is 435 square meters.

11-3

Areas of Regular Polygons and Circles

Areas of Regular Polygons In a regular polygon, the segment drawn from the center of the polygon perpendicular to the opposite side is called the **apothem**. In the figure at the right, $\overline{AP}$ is the apothem and $\overline{AR}$ is the radius of the circumscribed circle.



Area of a Regular Polygon	If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then $A = \frac{1}{2}Pa$.

Example 1 **Verify the formula $A = \frac{1}{2}Pa$ for the regular pentagon above.**

For $\triangle RAS$, the area is $A = \frac{1}{2}bh = \frac{1}{2}(RS)(AP)$. So the area of the pentagon is $A = 5\left(\frac{1}{2}\right)(RS)(AP)$. Substituting P for $5RS$ and substituting a for AP, then $A = \frac{1}{2}Pa$.

Example 2 **Find the area of regular pentagon $RSTUV$ above if its perimeter is 60 centimeters.**

First find the apothem.

The measure of central angle RAS is $\frac{360}{5}$ or 72. Therefore, $m\angle RAP = 36$. The perimeter is 60, so $RS = 12$ and $RP = 6$.

$$\tan \angle RAP = \frac{RP}{AP}$$

$$\tan 36^\circ = \frac{6}{AP}$$

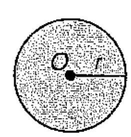
$$AP = \frac{6}{\tan 36^\circ}$$

$$\approx 8.26$$

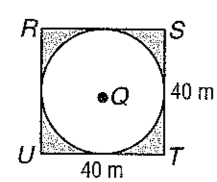
So, $A = \frac{1}{2}Pa = \frac{1}{2}60(8.26)$ or 247.8.

The area is about 248 square centimeters.

Areas of Circles As the number of sides of a regular polygon increases, the polygon gets closer and closer to a circle and the area of the polygon gets closer to the area of a circle.

Area of a Circle	If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$. 

Example **Circle Q is inscribed in square $RSTU$. Find the area of the shaded region.**



A side of the square is 40 meters, so the radius of the circle is 20 meters.

The shaded area is

Area of $RSTU$ − Area of circle Q

$= 40^2 - \pi r^2$

$= 1600 - 400\pi$

$\approx 1600 - 1256.6$

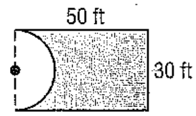
$= 343.4 \text{ m}^2$

11-4

Areas of Composite Figures

Composite Figures A composite figure is a figure that can be seprated into regions that are basic figures. To find the area of a composite figure separate the figure into basic figures of which we can find the area. The sum of the areas of the basic figures is the area of the figure.

Example 1 **Find the area of the composite figure.**



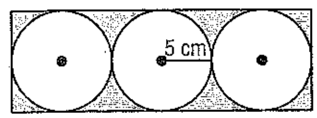
The figure is a rectangle minus one half of a circle. The radius of the circle is one half of 30 or 15.

$A = lw - \frac{1}{2}\pi r^2$

$= 50(30) - 0.5\pi(15)^2$

≈ 1146.6 or about 1147 ft^2

Example 2 **Find the area of the shaded region.**



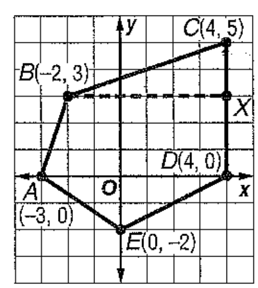
The dimensions of the rectangle are 10 centimeters and 30 centimeters. The area of the shaded region is

$(10)(30) - 3\pi(5^2) = 300 - 75\pi$

≈ 64.4 cm^2

Composite Figures on the Coordinate Plane To find the area of a composite figure on the coordinate plane, separate the figure into basic figures.

Example **Find the area of pentagon *ABCDE*.**



Draw $\overline{BX}$ between $B(-2, 3)$ and $X(4, 3)$ and draw $\overline{AD}$. The area of *ABCDE* is the sum of the areas of $\triangle BCX$, trapezoid *BXDA*, and $\triangle ADE$.

$A =$ area of $\triangle BCX$ + area of *BXDA* + area of $\triangle ADE$

$= \frac{1}{2}(2)(6) + \frac{1}{2}(3)(6 + 7) + \frac{1}{2}(2)(7)$

$= 6 + \frac{39}{2} + 7$

$= 32.5$ square units