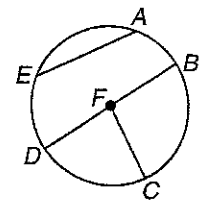
10-1 *Circles and Circumference*

Parts of Circles A **circle** consists of all points in a plane that are a given distance, called the **radius**, from a given point called the **center**.

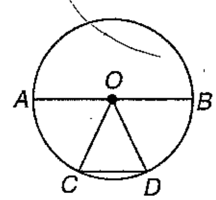


chord: $\overline{AE}$, $\overline{BD}$
radius: $\overline{FB}$, $\overline{FC}$, $\overline{FD}$
diameter: $\overline{BD}$

A segment or line can intersect a circle in several ways.

- A segment with endpoints that are the center of the circle and a point of the circle is a **radius**.
- A segment with endpoints that lie on the circle is a **chord**.
- A chord that contains the circle's center is a **diameter**.

Example



a. Name the circle.
The name of the circle is $\odot O$.

b. Name radii of the circle.
$\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$ are radii.

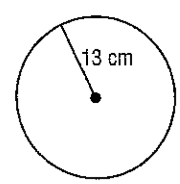
c. Name chords of the circle.
$\overline{AB}$ and $\overline{CD}$ are chords.

d. Name a diameter of the circle.
$\overline{AB}$ is a diameter.

Circumference The **circumference** of a circle is the distance around the circle.

Circumference	For a circumference of C units and a diameter of d units or a radius of r units, $C = \pi d$ or $C = 2\pi r$.

Example **Find the circumference of the circle to the nearest hundredth.**

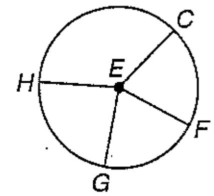


C	$= 2\pi r$	Circumference formula
	$= 2\pi(13)$	$r = 13$
	≈ 81.68	Use a calculator.

The circumference is about 81.68 centimeters.

10-2 *Measuring Angles and Arcs*

Angles and Arcs A **central angle** is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a **major arc** and a **minor arc**.

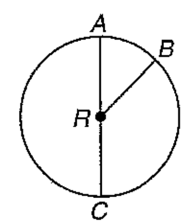


$\widehat{GF}$ is a minor arc.
$\widehat{CHG}$ is a major arc.
$\angle GEF$ is a central angle.

Here are some properties of central angles and arcs.

- The sum of the measures of the central angles of a circle with no interior points in common is 360. — $m\angle HEC + m\angle CEF + m\angle FEG + m\angle GEH = 360$
- The measure of a minor arc equals the measure of its central angle. — $m\widehat{CF} = m\angle CEF$
- The measure of a major arc is 360 minus the measure of the minor arc. — $m\widehat{CGF} = 360 - m\widehat{CF}$
- Two arcs are congruent if and only if their corresponding central angles are congruent. — $\widehat{CF} \cong \widehat{FG}$ if and only if $\angle CEF \cong \angle FEG$.
- The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. **(Arc Addition Postulate)** — $m\widehat{CF} + m\widehat{FG} = m\widehat{CG}$

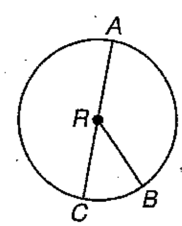
Example **In $\odot R$, $m\angle ARB = 42$ and $\overline{AC}$ is a diameter. Find $m\widehat{AB}$ and $m\widehat{ACB}$.**



$\angle ARB$ is a central angle and $m\angle ARB = 42$, so $m\widehat{AB} = 42$.
Thus $m\widehat{ACB} = 360 - 42$ or 318.

Arc Length An arc is part of a circle and its length is a part of the circumference of the circle.

Example **In $\odot R$, $m\angle ARB = 135$, $RB = 8$, and $\overline{AC}$ is a diameter. Find the length of $\widehat{AB}$.**



$m\angle ARB = 135$, so $m\widehat{AB} = 135$. Using the formula $C = 2\pi r$, the circumference is $2\pi(8)$ or 16π. To find the length of $\widehat{AB}$, write a proportion to compare each part to its whole.

$$\frac{\text{length of } \widehat{AB}}{\text{circumference}} = \frac{\text{degree measure of arc}}{\text{degree measure of circle}} \quad \text{Proportion}$$

$$\frac{\ell}{16\pi} = \frac{135}{360} \quad \text{Substitution}$$

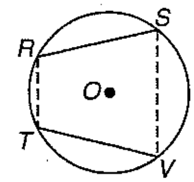
$$\ell = \frac{(16\pi)(135)}{360} \quad \text{Multiply each side by } 16\pi.$$

$$= 6\pi \quad \text{Simplify.}$$

The length of $\widehat{AB}$ is 6π or about 18.85 units.

10-3 *Arcs and Chords*

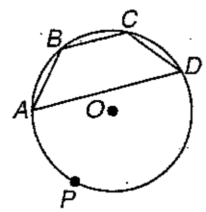
Arcs and Chords Points on a circle determine both chords and arcs. Several properties are related to points on a circle.



$\overset{\frown}{RS} \cong \overset{\frown}{TV}$ if and only if $\overline{RS} \cong \overline{TV}$.
RSVT is inscribed in $\odot O$.
$\odot O$ is circumscribed about *RSVT*.

- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- If all the vertices of a polygon lie on a circle, the polygon is said to be **inscribed** in the circle and the circle is **circumscribed** about the polygon.

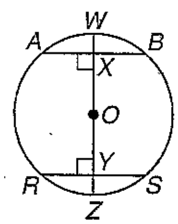
Example **Trapezoid *ABCD* is inscribed in $\odot O$. If $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and $m\overset{\frown}{BC} = 50$, what is $m\overset{\frown}{APD}$?**



Chords $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ are congruent, so $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, and $\overset{\frown}{CD}$ are congruent. $m\overset{\frown}{BC} = 50$, so $m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{CD} = 50 + 50 + 50 = 150$. Then $m\overset{\frown}{APD} = 360 - 150$ or 210.

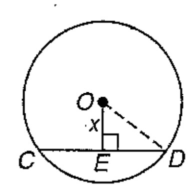
Diameters and Chords

- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



If $\overline{WZ} \perp \overline{AB}$, then $\overline{AX} \cong \overline{XB}$ and $\overset{\frown}{AW} \cong \overset{\frown}{WB}$.
If $OX = OY$, then $\overline{AB} \cong \overline{RS}$.
If $\overline{AB} \cong \overline{RS}$, then $\overline{AB}$ and $\overline{RS}$ are equidistant from point *O*.

Example **In $\odot O$, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find x.**



A diameter or radius perpendicular to a chord bisects the chord, so *ED* is half of *CD*.

$$ED = \frac{1}{2}(24)$$
$$= 12$$

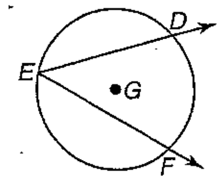
Use the Pythagorean Theorem to find x in $\triangle OED$.

$(OE)^2 + (ED)^2 = (OD)^2$	Pythagorean Theorem
$x^2 + 12^2 = 15^2$	Substitution
$x^2 + 144 = 225$	Multiply.
$x^2 = 81$	Subtract 144 from each side.
$x = 9$	Take the square root of each side.

10-4 *Inscribed Angles*

Inscribed Angles

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. In $\odot G$, inscribed $\angle DEF$ **intercepts** $\widehat{DF}$.



$m\angle DEF = \frac{1}{2}m\widehat{DF}$

Inscribed Angle Theorem	If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.

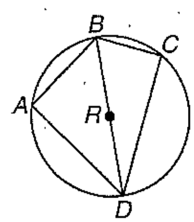
Example **In $\odot G$ above, $m\widehat{DF} = 90$. Find $m\angle DEF$.**

$\angle DEF$ is an inscribed angle so its measure is half of the intercepted arc.

$$m\angle DEF = \frac{1}{2}m\widehat{DF}$$
$$= \frac{1}{2}(90) \text{ or } 45$$

Angles of Inscribed Polygons

An **inscribed polygon** is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.



- If an angle of an inscribed polygon intercepts a semicircle, the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

If $\widehat{BCD}$ is a semicircle, then $m\angle BCD = 90$.

For inscribed quadrilateral $ABCD$,
$m\angle A + m\angle C = 180$ and
$m\angle ABC + m\angle ADC = 180$.

Example **In $\odot R$ above, $BC = 3$ and $BD = 5$. Find each measure.**

a. $m\angle C$

$\angle C$ intercepts a semicircle. Therefore $\angle C$ is a right angle and $m\angle C = 90$.

b. CD

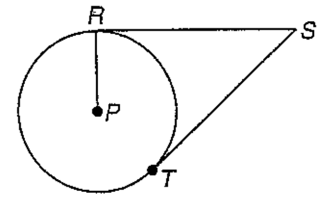
$\triangle BCD$ is a right triangle, so use the Pythagorean Theorem to find CD.

$$(CD)^2 + (BC)^2 = (BD)^2$$
$$(CD)^2 + 3^2 = 5^2$$
$$(CD)^2 = 25 - 9$$
$$(CD)^2 = 16$$
$$CD = 4$$

10-5 *Tangents*

Tangents A tangent to a circle intersects the circle in exactly one point, called the **point of tangency**. There are three important relationships involving tangents.

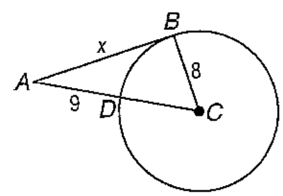
- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
- If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.



$\overline{RP} \perp \overline{SR}$ if and only if $\overline{SR}$ is tangent to $\odot P$.

If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

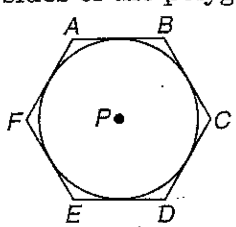
Example **$\overline{AB}$ is tangent to $\odot C$. Find x.**



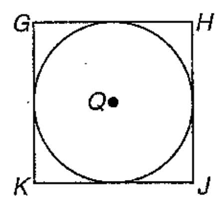
$\overline{AB}$ is tangent to $\odot C$, so $\overline{AB}$ is perpendicular to radius $\overline{BC}$. $\overline{CD}$ is a radius, so $CD = 8$ and $AC = 9 + 8$ or 17. Use the Pythagorean Theorem with right $\triangle ABC$.

$(AB)^2 + (BC)^2 = (AC)^2$	Pythagorean Theorem
$x^2 + 8^2 = 17^2$	Substitution
$x^2 + 64 = 289$	Multiply.
$x^2 = 225$	Subtract 64 from each side.
$x = 15$	Take the positive square root of each side.

Circumscribed Polygons When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.

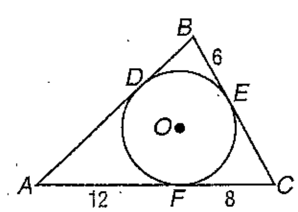


Hexagon *ABCDEF* is circumscribed about $\odot P$. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$ are tangent to $\odot P$.



Square *GHJK* is circumscribed about $\odot Q$. $\overline{GH}$, $\overline{JH}$, $\overline{JK}$, and $\overline{KG}$ are tangent to $\odot Q$.

Example **$\triangle ABC$ is circumscribed about $\odot O$. Find the perimeter of $\triangle ABC$.**



$\triangle ABC$ is circumscribed about $\odot O$, so points D, E, and F are points of tangency. Therefore $AD = AF$, $BE = BD$, and $CF = CE$.

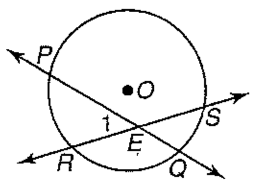
$P = AD + AF + BE + BD + CF + CE$
$= 12 + 12 + 6 + 6 + 8 + 8$
$= 52$

The perimeter is 52 units.

10-6 *Secants, Tangents, and Angle Measures*

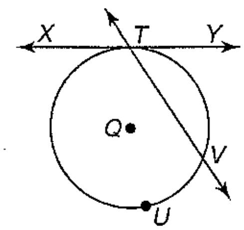
Intersections On or Inside a Circle A line that intersects a circle in exactly two points is called a **secant.** The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.



$m\angle 1 = \frac{1}{2}(m\widehat{PR} + m\widehat{QS})$

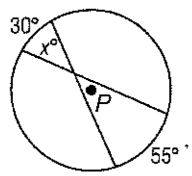
- If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.



$m\angle XTV = \frac{1}{2}m\widehat{TUV}$

$m\angle YTV = \frac{1}{2}m\widehat{TV}$

Example 1 **Find *x*.**



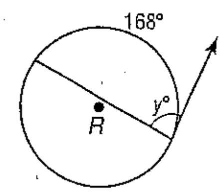
The two secants intersect inside the circle, so x is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

$x = \frac{1}{2}(30 + 55)$

$= \frac{1}{2}(85)$

$= 42.5$

Example 2 **Find *y*.**



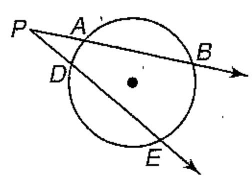
The secant and the tangent intersect at the point of tangency, so the measure of the angle is one-half the measure of its intercepted arc.

$y = \frac{1}{2}(168)$

$= 84$

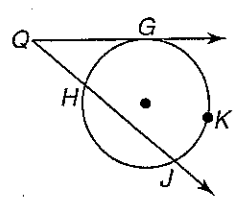
Intersections Outside a Circle If secants and tangents intersect outside a circle, they form an angle whose measure is related to the intercepted arcs.

If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.



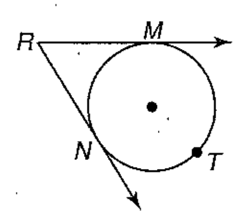
$\overrightarrow{PB}$ and $\overrightarrow{PE}$ are secants.

$m\angle P = \frac{1}{2}(m\widehat{BE} - m\widehat{AD})$



$\overrightarrow{QG}$ is a tangent. $\overrightarrow{QJ}$ is a secant.

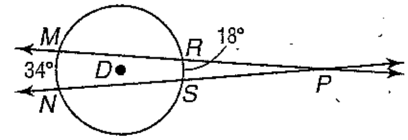
$m\angle Q = \frac{1}{2}(m\widehat{GKJ} - m\widehat{GH})$



$\overrightarrow{RM}$ and $\overrightarrow{RN}$ are tangents.

$m\angle R = \frac{1}{2}(m\widehat{MTN} - m\widehat{MN})$

Example **Find $m\angle MPN$.**



$\angle MPN$ is formed by two secants that intersect in the exterior of a circle.

$m\angle MPN = \frac{1}{2}(m\widehat{MN} - m\widehat{RS})$

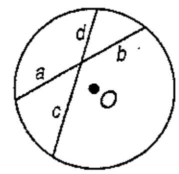
$= \frac{1}{2}(34 - 18)$

$= \frac{1}{2}(16)$ or 8

The measure of the angle is 8.

10-7 *Special Segments in a Circle*

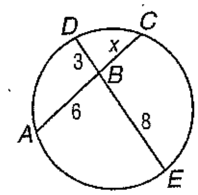
Segments Intersecting Inside a Circle If two chords intersect in a circle, then the products of the measures of the chords are equal.



$a \cdot b = c \cdot d$

Example **Find x.**

The two chords intersect inside the circle, so the products $AB \cdot BC$ and $EB \cdot BD$ are equal.



$AB \cdot BC = EB \cdot BD$

$AB \cdot BC = EB \cdot BD$

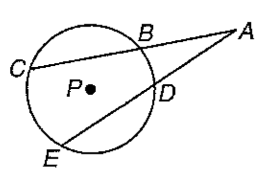
$6 \cdot x = 8 \cdot 3$ Substitution

$6x = 24$ Simplify.

$x = 4$ Divide each side by 6.

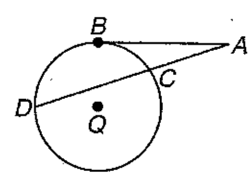
Segments Intersecting Outside a Circle If secants and tangents intersect outside a circle, then two products are equal.

- If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



$\overline{AC}$ and $\overline{AE}$ are secant segments.
$\overline{AB}$ and $\overline{AD}$ are external secant segments.
$AC \cdot AB = AE \cdot AD$

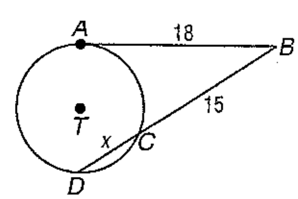
- If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



$\overline{AB}$ is a tangent segment.
$\overline{AD}$ is a secant segment.
$\overline{AC}$ is an external secant segment.
$(AB)^2 = AD \cdot AC$

Example **Find x to the nearest tenth.**

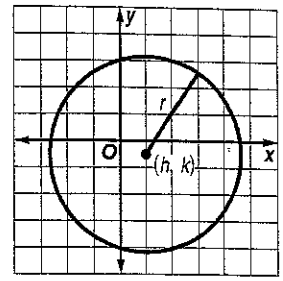
The tangent segment is $\overline{AB}$, the secant segment is $\overline{BD}$, and the external secant segment is $\overline{BC}$.



$(AB)^2 = BC \cdot BD$
$(18)^2 = 15(15 + x)$
$324 = 225 + 15x$
$99 = 15x$
$6.6 = x$

10-8 *Equations of Circles*

Equation of a Circle A **circle** is the locus of points in a plane equidistant from a given point. You can use this definition to write an equation of a circle.



Standard Equation of a Circle An equation for a circle with center at (h, k) and a radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example **Write an equation for a circle with center (−1, 3) and radius 6.**

Use the formula $(x - h)^2 + (y - k)^2 = r^2$ with $h = -1$, $k = 3$, and $r = 6$.

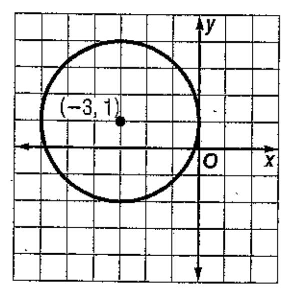
$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - (-1))^2 + (y - 3)^2 = 6^2$ Substitution

$(x + 1)^2 + (y - 3)^2 = 36$ Simplify.

Graph Circles If you are given an equation of a circle, you can find information to help you graph the circle.

Example **Graph $(x + 3)^2 + (y - 1)^2 = 9$.**



Use the parts of the equation to find (h, k) and r.

$(x - h)^2 + (y - k)^2 = r^2$

$(x - h)^2 = (x + 3)^2$	$(y - k)^2 = (y - 1)^2$	$r^2 = 9$
$x - h = x + 3$	$y - k = y - 1$	$r = 3$
$-h = 3$	$-k = -1$	
$h = -3$	$k = 1$	

The center is at $(-3, 1)$ and the radius is 3. Graph the center. Use a compass set at a radius of 3 grid squares to draw the circle.